



Mark Scheme (Results)

Summer 2024

Pearson Edexcel GCE In A level Further
Mathematics
Paper 9FM0/3C

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award **zero marks if the candidate's response is not worthy of credit** according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application **of the mark scheme to a candidate's response, the team leader must be consulted.**
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol \surd will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - \square The second mark is dependent on gaining the first mark
4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $g = 9.8$ should be given to 2 or 3 SF.
- Use of $g = 9.81$ should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

M(A)	Taking moments about A.
N2L	Newton's Second Law (Equation of Motion)
NEL	Newton's Experimental Law (Newton's Law of Impact)
HL	Hooke's Law
SHM	Simple harmonic motion
PCLM	Principle of conservation of linear momentum
RHS, LHS	Right hand side, left hand side.

Question	Scheme	Marks	AOs
1(a)	Form an expression for total KE	M1	3.1a
	$\frac{1}{2} \times 3 \times (3^2 + (-1)^2) + \frac{1}{2} \times 2 \times ((-6)^2 + 2^2)$	A1	1.1b
	= 55 (J)	A1	1.1b
		(3)	
1(b)	Use of change in momentum	M1	3.1a
	$3 \left((-2\mathbf{i} + \frac{2}{3}\mathbf{j}) - (3\mathbf{i} - \mathbf{j}) \right)$	A1	1.1b
	= $(-15\mathbf{i} + 5\mathbf{j})$ (N s)	A1	1.1b
		(3)	
1(c)	Either: Use impulse on B to form equation	M1	3.1a
	$(15\mathbf{i} - 5\mathbf{j}) = 2(\mathbf{v}_B - (-6\mathbf{i} + 2\mathbf{j}))$	A1	1.1b
	$\mathbf{v}_B = (1.5\mathbf{i} - 0.5\mathbf{j})$ (m s ⁻¹)	A1	1.1b
	Or: Use CLM to form equation	M1	3.1a
	$3(3\mathbf{i} - \mathbf{j}) + 2(-6\mathbf{i} + 2\mathbf{j}) = 3(-2\mathbf{i} + \frac{2}{3}\mathbf{j}) + 2\mathbf{v}_B$	A1	1.1b
	$\mathbf{v}_B = (1.5\mathbf{i} - 0.5\mathbf{j})$ (m s ⁻¹)	A1	1.1b
		(3)	
(9 marks)			
Notes: Ignore units in final answers			
(a)			
M1	Form an expression for total KE, correct number of terms and dimensionally correct. Must add 2 KE terms of the correct structure. Allow missing minus signs on components when squaring. M0 for use of $(3\mathbf{i} - \mathbf{j})^2$ unless recovered. M0 if KE terms are never added together. M0 for using $\frac{1}{2}mv$ unless the correct KE formula is also stated. (Note that $\frac{1}{2}mv$ is not dimensionally correct)		
A1	Correct unsimplified expression for the total KE. If the 2 KE terms are incorrect when the total is formed, this is A0.		
A1	Cao		
(b)	N.B. Only penalise use of column vectors in final answers once across (b) and (c). Penalise the first occurrence. Allow column vectors throughout working in (b) and (c).		

M1	Use of change in momentum, dimensionally correct (mass \times velocity). Must subtract momenta but condone in wrong order. Must use mass of 3kg with velocities of <i>A</i> or mass of 2kg with velocities of <i>B</i> . M0 if speeds are used.
A1	Correct unsimplified expression for impulse
A1	Correct answer in i and j form. Penalise first occurrence if answer is given as a column vector. ISW if they continue and find the magnitude.
(c)	
M1	Either: Use of impulse-momentum equation with their Impulse. Must subtract momenta (mass \times velocity) but condone in wrong order. Condone sign error on LHS.
A1	Correct unsimplified equation
A1	Correct answer in i and j form. Penalise first occurrence if answer is given as a column vector.
M1	Or: Use of CLM equation with correct number of terms, condone sign errors. Masses and velocities must be paired correctly. M0 if speeds are used.
A1	Correct unsimplified equation
A1	Correct answer in i and j form. Penalise first occurrence if answer is given as a column vector.

Question	Scheme	Marks	AOs
2(a)	Resolve perpendicular to the plane and use $F = \mu R$	M1	3.1a
	$\frac{1}{7}mg\cos\theta$	A1	1.1b
	$\frac{1}{7}mg \times \frac{4}{5} = \frac{4mg}{35} *$ or $\frac{4}{35}mg *$	A1*	1.1b
		(3)	
2(b)	Use of work-energy principle	M1	3.3
	$\frac{4mgd}{35} = \frac{1}{2}m \times 2ag - mgd \sin\theta$	A1	1.1b
		A1	1.1b
	$d = \frac{7a}{5}$ oe	A1	1.1b
		(4)	
(7 marks)			
Notes:			
(a)			
M1	Resolve perpendicular to the plane to find an expression for R and use μR . Condone sin/cos confusion on weight component. All required terms present and no extras. Dimensionally correct.		
A1	Correct unsimplified expression for friction. Allow with $\cos\theta$ or $\frac{4}{5}$		
A1*	Given answer correctly obtained. Working out must include both $\frac{1}{7}$ and $\frac{4}{5}$ in the same line before reaching the given answer.		
(b)			
M1	Use of work-energy principle with correct number of terms: 1 work, 1 KE, 1 GPE. Condone \pm sign errors on terms. All required terms present and no extras. Resolve only when required and condone cos/sin confusion for the method mark. Must use given answer from (a) in work term. M0 if Friction is not multiplied by distance (dimensionally incorrect) M0 if a term is missing. M0 for incorrect use of trig eg $d \tan\theta$, $\frac{d}{\sin\theta}$, $\frac{d}{\cos\theta}$...		
A1	Correct equation with at most one error		
A1	Correct equation		
A1	Correct answer for d . Any equivalent fraction or decimal multiple of a		

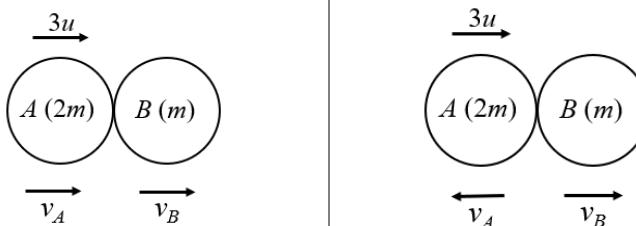
Question	Scheme	Marks	AOs
3(a)	Use $F = \frac{1000P}{v}$ where $v = \frac{72000}{3600}$ ($= 20$)	M1	3.3
	Use equation of motion: $F - 900 = 0$ to give equation in P only: Eg $\frac{1000P}{20} = 900$	M1	3.1b
	$P = 18$	A1	1.1b
		(3)	
3(b)	Use equation of motion for car and power equation to give equation in a only	M1	3.1b
	$\frac{30000}{10} - 1000g \sin \alpha - 20 \times 10 = 1000a$ o.e.	A1	1.1b
		A1	1.1b
	$a = 2.4$	A1	1.1b
		(4)	
3(c)	$F = \frac{30\,000}{U}$	M1	3.4
	Equation of motion for car	M1	3.1b
	$F - 1000g \sin \alpha - 20U = 0$	A1	1.1b
	Correct equation in U only oe Eg <ul style="list-style-type: none"> $\frac{30000}{U} - 1000g \left(\frac{2}{49}\right) - 20U = 0$ $U^2 + 20U - 1500 = 0$ 	A1	1.1b
	$U = 30$	A1	1.1b
		(5)	

(12 marks)

Notes:

(a)	
M1	Use of $P = Fv$, condone $\frac{P}{72}$ or $\frac{P}{20}$
M1	Use equation of motion to give equation in P only. Condone use of P instead of $1000P$ and condone 72 instead of 20 for method mark. $\frac{1000P}{20} = 900$ or $\frac{P}{20} = 900$
A1	Cao Allow $P = 18\,000$ leading to a final answer of $P = 18$. Ignore units.
(b)	NB: Only penalise use of $g = 9.81$ once per question

M1	Use equation of motion for the car and power equation to give a dimensionally correct equation in a only. All required terms present and no extras, resolving only where necessary. Condone \pm sign errors and cos/sin confusion. M0 if a term is missing or if weight is not resolved.
A1	Correct equation in a only with at most one error.
A1	Correct equation in a only
A1	Answer of 2.4 only A0 for $\frac{12}{5}$ (when using $g = 9.8$, answers must be rounded to 2/3sf, not given in exact form) A0 for use of $g = 9.81$
(c)	
M1	Use of $P = Fv$, condone incorrect number of zeros M0 if using speed of 10 from part (b)
M1	Equation of motion for the car: all required terms present and no extras, resolving only where necessary, dimensionally correct. Condone \pm sign errors and cos/sin confusion. M0 if a term is missing or if weight is not resolved. M0 if using resistance of 200 from part (b)
A1	A correct unsimplified equation, F does not need to be substituted, sin/cos does not need to be substituted.
A1	A correct equation in U only.
A1	Answer of 30 only

Question	Scheme		Marks	AOs
4(a)				
	CLM:		M1	3.1a
	$2m \times 3u = 2mv_A + mv_B$ $(6u = 2v_A + v_B)$	$2m \times 3u = -2mv_A + mv_B$ $(6u = -2v_A + v_B)$	A1	1.1b
	Impact Law:		M1	3.4
	$3ue = -v_A + v_B$	$3ue = v_A + v_B$	A1	1.1b
	Solve for v_B		M1	2.1
	$(v_B =) 2u(1+e) *$		A1*	2.2a
			(6)	
4(b)	Solve for v_A		M1	3.1a
	$v_A = u(2-e)$	$v_A = u(e-2)$	A1	1.1b
	Complete and correct explanation: $0 \leq e \leq 1 \Rightarrow v_A > 0$ $\Rightarrow A$ continues to move towards the wall $\Rightarrow A$ will collide again with B .	Complete and correct explanation: $0 \leq e \leq 1 \Rightarrow v_A < 0$ $\Rightarrow A$ continues to move towards the wall $\Rightarrow A$ will collide again with B .	A1	2.4
			(3)	
4(c)	Rebound speed or velocity of $B = \pm \frac{1}{2} \times 2u(1+e)$		B1	3.4
	$\pm m[-u(1+e) - 2u(1+e)]$		M1	3.1a
	$3(1+e)mu$		A1	1.1b
			(3)	
4(d)	Attempt at KE loss of B		M1	3.4
	$\frac{1}{2} m [(2u(1+e))^2 - (u(1+e))^2]$		A1	3.1a
	$\frac{3mu^2(1+e)^2}{2}$		A1	1.1b
			(3)	
	(15 marks)			
	Notes:			

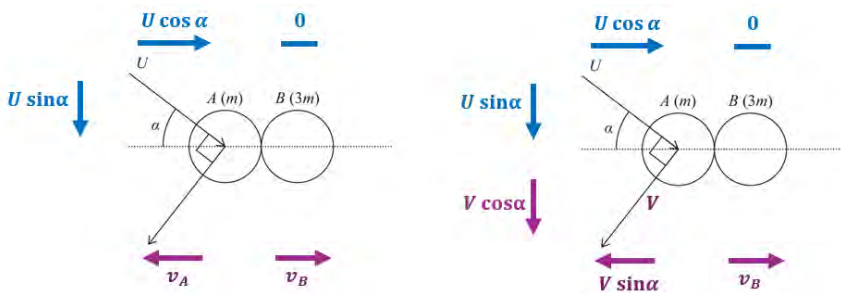
(a)	
M1	Use of CLM, all terms required, dimensionally correct (mass \times velocity in each term). Mass and velocity paired correctly. Condone sign errors on velocities. Condone consistent extra g and/or consistent missing m (in every term).
A1	Correct unsimplified equation.
M1	Correct use of Impact Law, dimensionally correct, condone sign errors on velocity. M0 if separation and approach are on the wrong sides.
A1	Correct unsimplified equation, the direction of A must be consistent with their CLM.
M1	Use their <i>correctly formed</i> equations to solve for v_B
A1*	Given answer correctly obtained and exactly as printed. Working should include an equation in v_B only <i>before</i> reaching the given answer.
(b)	
M1	Use given answer in (a) to solve for v_A . If seen in (a) it must be used in (b) to earn this mark.
A1	Correct expression seen for velocity of A after impact.
A1*	Correct and complete explanation with no incorrect statements. Must include all of: <ul style="list-style-type: none"> $0 < e \leq 1$ or $0 \leq e \leq 1$ or $0 \leq e < 1$ or 'for all e' $v_A > 0$ or $v_A < 0$ (must be correct for their v_A) Must refer to the wall. A continues to move with unchanged direction or towards the wall (eg do not accept descriptions for direction of travel as 'to the right' or similar) Conclude second collision between A and B Note: A0* for explanations that rely on comparing speed of v_B and v_A
(c)	
B1	Correct unsimplified expression for rebound speed or velocity of B , may be seen on diagram or elsewhere in working. Accept positive or negative.
M1	Attempt to find difference in momenta, dimensionally correct (mass \times velocity). Must use given answer from (a) for V_b . Condone use of e or e' in place of $\frac{1}{2}$
A1	Correct answer, must be positive. Accept any equivalent form that is either factorised or reduced to 2 terms eg $3(1+e)mu$, $3mu + 3emu$
(d)	
M1	Attempt at KE loss of B : clear attempt at a difference in KE of B before and after impact with the wall. Must use velocity of B from (a). Condone use of e or e' in place of $\frac{1}{2}$. Dimensionally correct, condone subtraction either way round.
A1	Correct unsimplified expression for KE loss of B
A1	Any equivalent factorised form eg $\frac{3mu^2(1+e)^2}{2}$, $\frac{3mu^2(1+2e+e^2)}{2}$, or

Question	Scheme	Marks	AOs
5.	GPE from E to instantaneous rest eg mgx , $mg(d - a)$	M1	1.1b
	Use of conservation of energy principle	M1	3.1a
	For example,	A1	1.1b
	<ul style="list-style-type: none"> $\frac{1}{2}m\frac{9ag}{4} + mgx = \frac{2mg(a+x)^2}{4a} - \frac{2mga^2}{4a}$ oe $\frac{1}{2}m\frac{9ag}{4} + mg(d - a) = \frac{2mgd^2}{2(2a)} - \frac{2mga^2}{2(2a)}$ oe 	A1	1.1b
	Accept equivalent rearrangements in one unknown length		
	$x = \frac{3a}{2}$ or $d = \frac{5a}{2}$	A1	1.1b
	Use of EPE formula	M1	2.1
	$\frac{25mga}{8}$	A1	2.2a
		(7)	
(7 marks)			
Notes:			
M1	Use of GPE for unknown distance from E to instantaneous rest. May be implied by a difference of 2 GPE terms.		
M1	Use of conservation of energy principle to form an equation with one KE term, one GPE term, two EPE terms. For the method mark only we will, condone only one EPE term. However, all terms are required for A marks. Dimensionally correct energy equation (energy terms must have the correct structure). Note there are common rearrangements. Eg Initial Energy = Final Energy, Energy Loss = Energy Gain oe Condone \pm sign errors. M0 for use of <i>suvat</i> . M0 if 'E' or similar is used to represent unknown EPE unless recovered.		
A1	All 4 terms present in an energy equation with one unknown length. At most one error. A0 if an energy term is missing.		
A1	Fully correct equation in one unknown length.		
A1	cao		
M1	Use of EPE formula at least once. May be seen here or in earlier working. EPE must have the form $\frac{\lambda x^2}{ka}$ where λ is modulus of elasticity, k is a constant and x is extension.		
A1	Correct answer		

Question	Scheme	Marks	AOs
6(a)	<p>$\mathbf{v} = (\mathbf{i} + 3\mathbf{j})$ $\mathbf{v} \cdot \hat{\mathbf{i}} = \frac{9}{5}$</p> <p>$\mathbf{u} = (4\mathbf{i} - \mathbf{j})$ $\mathbf{u} \cdot \hat{\mathbf{i}} = -\frac{16}{5}$</p> <p>Impulse // to $(-3\mathbf{i} + 4\mathbf{j})$, $\hat{\mathbf{i}} = \frac{1}{5}(-3\mathbf{i} + 4\mathbf{j})$</p>		
	Complete method to find direction of normal (perpendicular to wall) $(\pm m) ((\mathbf{i} + 3\mathbf{j}) - (4\mathbf{i} - \mathbf{j}))$	M1	2.1
	$(-3\mathbf{i} + 4\mathbf{j})$ or any parallel vector	A1	1.1b
	Resolve both velocities parallel to impulse	M1	3.1a
	Approach: $(4\mathbf{i} - \mathbf{j}) \cdot \left(\frac{1}{5}\right)(-3\mathbf{i} + 4\mathbf{j}) = -\frac{16}{5}$ N.B. $\frac{1}{5}$ is not required	A1	1.1b
	Separation: $(\mathbf{i} + 3\mathbf{j}) \cdot \left(\frac{1}{5}\right)(-3\mathbf{i} + 4\mathbf{j}) = \frac{9}{5}$ N.B. $\frac{1}{5}$ is not required		
	$e = \text{separation speed} / \text{approach speed}$	DM1	3.3
	$e = \frac{9}{16}$ (0.56 or better)	A1	1.1b
	(6)		
6(b)	<p>$\mathbf{v} = (\mathbf{i} + 3\mathbf{j})$ $\mathbf{w} = (\mathbf{i} - \mathbf{j})$</p> <p>Wall // to \mathbf{i}</p> <p>$e = \frac{1}{3}$</p> <p>$\frac{1}{3} \times 3 = 1$</p>		
	Complete method to find both components of rebound velocity	M1	2.1
	<ul style="list-style-type: none"> // to wall (\mathbf{i}), unchanged Perp (\mathbf{j}), $\frac{1}{3} \times 3$ 		
	Both velocity components correct, may be given if seen on a diagram.	A1	1.1b
	Complete method to find the angle of deflection, α° Eg <ul style="list-style-type: none"> Using trig $\alpha^\circ = \theta + \phi = \tan^{-1}\left(\frac{3}{1}\right) + \tan^{-1}\left(\frac{1}{1}\right)$ Using scalar product $\alpha^\circ = \cos^{-1}\left(\frac{(\mathbf{i} + 3\mathbf{j}) \cdot (\mathbf{i} - \mathbf{j})}{\sqrt{1^2 + 3^2} \sqrt{1^2 + 1^2}}\right) = \cos^{-1}\left(\frac{-2}{\sqrt{10} \sqrt{2}}\right)$ Note: $\tan \phi = \frac{1}{3} \tan \theta$ may be used in working	DM1	3.1a
	$\alpha = 117$ (nearest whole number)	A1	1.1b

		(4)	
(10 marks)			
Notes: Allow column vectors throughout the question			
(a)			
M1	<p>Complete method to find the direction of the normal (perpendicular to the wall). Eg</p> <ul style="list-style-type: none"> Main scheme: use of impulse equation with difference in momenta, m is not required so may see only the difference in velocities. Find the direction of the wall first by equating parallel components then find a perpendicular direction using the scalar product or otherwise. Eg $(4\mathbf{i} - \mathbf{j}) \cdot (x\mathbf{i} + y\mathbf{j}) = (\mathbf{i} + 3\mathbf{j}) \cdot (x\mathbf{i} + y\mathbf{j}) \Rightarrow 3x = 4y$ Wall direction $\parallel (4\mathbf{i} + 3\mathbf{j}) \Rightarrow$ Impulse direction $\parallel (-3\mathbf{i} + 4\mathbf{j})$ 		
A1	Any correct vector parallel to impulse $(-3\mathbf{i} + 4\mathbf{j})$		
M1	<p>Complete method to resolve both velocities parallel to impulse</p> <ul style="list-style-type: none"> Using the scalar product (main scheme) Using trig: velocity components from shown on diagram <p style="text-align: center;">where $\tan \theta = \frac{3}{4}$ $\tan \alpha = \frac{1}{4}$ $\tan \beta = \frac{3}{1}$</p> <ul style="list-style-type: none"> Using rotation matrix : velocity components shown on RHS of matrix equation <div style="display: flex; align-items: center;"> <div style="margin-right: 20px;"> $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 3.2 \\ 2.6 \end{pmatrix}$ $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -1.8 \\ 2.6 \end{pmatrix}$ </div> </div>		
A1	Both correct velocity components parallel to impulse, allow positive or negative values in both cases.		
DM1	Dependent on previous M. Use of Impact Law parallel to impulse. Ratio of speeds correct way up (separation \div approach)		
A1	Correct answer (0.56 or better)		
(b)			

M1	Method to find velocity after impact with second wall: i component unchanged, j component: $\frac{1}{3}$ (3) . Condone sign errors.
A1	Correct velocity components after impact, may be given if seen on a diagram.
DM1	Complete method to find the angle of deflection for their velocity after second impact. Dependent on previous M.
A1	Correct answer, must be rounded to nearest degree.

Question	Scheme		Marks	AOs
				
7(a)	CLM along line of centres		M1	3.1b
	$mU \cos \alpha = -mv_A + 3mv_B$	$mU \cos \alpha = -mV \sin \alpha + 3mv_B$	A1	1.1b
	Impact Law used along line of centres		M1	3.3
	$eU \cos \alpha = v_A + v_B$	$eU \cos \alpha = V \sin \alpha + v_B$	A1	1.1b
	Solve for v_B		DM1	2.1
	$\frac{1}{4}(1+e)U \cos \alpha$ *		A1*	2.2a
			(6)	
7(b)	Unsimplified expression for v_A seen	Unsimplified expression for $V \sin \alpha$ seen	M1	2.1
	$= \frac{1}{4}(3e-1)U \cos \alpha$ oe seen		A1	1.1b
	Solve $v_A > 0$ to find an inequality for e	Solve $V \sin \alpha > 0$ to find an inequality for e	M1	3.1b
	Correct and complete reasoning leading to $e > \frac{1}{3}$ *. Must include: correct inequality for v_A , $0^\circ < \alpha < 90^\circ$ or α is acute, $\cos \alpha > 0$.		A1*	2.2a
			(4)	
7(c)	Velocity component of A, perpendicular to the line of centres, after collision = $U \sin \alpha$		B1	3.3
	Use of 90° deflection: $\tan \alpha = \frac{\frac{1}{4}(3e-1)U \cos \alpha}{U \sin \alpha}$		M1	2.1
	Correct equation in e and $\tan \alpha$ $\tan^2 \alpha = \frac{1}{4}(3e-1)$ oe		A1	1.1b

	$e \leq 1 \Rightarrow \tan^2 \alpha \leq \frac{1}{2}$	DM1	3.1b
	$0 < \tan \alpha \leq \frac{1}{\sqrt{2}} \quad \text{since} \quad 0^\circ < \alpha \leq 90^\circ \quad *$	A1*	2.2a
		(5)	
(15 marks)			
Notes:			
(a)			
M1	Use of CLM along the line of centres with mass and velocities paired correctly. Correct no. of terms, condone sin/cos confusion and sign errors on velocities. Allow consistent missing m 's and/or consistent extra g 's (every term).		
A1	Correct unsimplified equation		
M1	Use of Impact Law along the line of centres. Correct no. of terms, condone sin/cos confusion and sign errors but e must be on the correct side of the equation		
A1	Correct unsimplified equation (signs consistent with CLM equation)		
DM1	Dependent on both previous M's. Solve for their v_B		
A1*	Given answer correctly obtained and EXACTLY as printed. Working should include an equation in v_B only before reaching given answer.		
(b)			
M1	Use the given expression for v_B to find an unsimplified expression for v_A or $V \sin \alpha$ Note: If seen in (a) it must be used in (b) to achieve the mark.		
A1	Correct unsimplified expression for v_A or $V \sin \alpha$ seen		
M1	Solve $v_A > 0$ or $V \sin \alpha > 0$ to find an inequality for e		
A1*	Given answer complete and correctly obtained with no incorrect statements. Complete explanation must include: <ul style="list-style-type: none"> • correct inequality for v_A • $0^\circ < \alpha < 90^\circ$ or α is acute • $\cos \alpha > 0$ Leading to $e > \frac{1}{3}$		
(c)			
B1	Velocity component of A , perpendicular to the line of centres, after collision is $U \sin \alpha$ May be seen labelled on the diagram or written as an expression early on in their working. Does not need to be used here.		
M1	Use of perpendicular deflection to form an equation in α and e (and U). Allow reciprocal. Alternative methods include: <ul style="list-style-type: none"> • scalar product of vectors, eg $\begin{pmatrix} U \cos \alpha \\ U \sin \alpha \end{pmatrix} \cdot \begin{pmatrix} -v_A \\ U \sin \alpha \end{pmatrix} = 0$ with v_A substituted 		

	<ul style="list-style-type: none"> • $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$ with $\alpha + \beta = 90 \Rightarrow 1 - \tan \alpha \tan \beta = 0 \Rightarrow \dots$
A1	<p>A correct equation in $\tan \alpha$ and e only</p> <p>eg $\tan^2 \alpha = \frac{1}{4}(3e - 1)$ or $e = \frac{4 \tan^2 \alpha + 1}{3}$ oe</p>
DM1	<p>Dependent on previous M. Clear explanation using $e \leq 1$ to form an inequality in $\tan \alpha$. Condone, use of $\max e = 1 \Rightarrow \max \tan^2 \alpha = \dots$</p>
A1*	<p>Given answer obtained from correct and complete working with no incorrect statements. Complete explanation must justify both sides of inequality using both $e \leq 1$ and $0^\circ < \alpha < 90^\circ$ (allow α is acute)</p>

